nominally linear pitching moment. Thus,

$$\frac{\delta\alpha}{\alpha_0} = \frac{\pi}{\Omega d}\frac{\rho Ad}{4m}\left[C_{L_\alpha} - C_D - \frac{md^2}{I}\,(C_{m_q} + C_{m\dot{\alpha}})\right] - \frac{1}{C_{m_\alpha}{\alpha_0}^2}\int_{-\alpha_0}^{+\alpha_0} C_m d\alpha \qquad (1)$$

The last term in the equation is the contribution due to hysteresis. The real dynamic stability coefficient, given by Eq. (1) when a hysteresis is present, is as follows:

$$(C_{m_q} + C_{m\dot{\alpha}})_{\text{real}} = -\frac{I}{md^2}\left[\left(\frac{\delta\alpha}{\alpha_0} + \frac{1}{C_{m_\alpha}{\alpha_0}^2}\int_{-\alpha_0}^{+\alpha_0} C_m d\alpha\right)\frac{\Omega d}{\pi}\frac{4m}{\rho Ad} - C_{L_\alpha} + C_D\right] \qquad (2)$$

However, if a test is performed where the presence of a hysteresis is not known, its influence will be lumped into an apparent or test dynamic stability coefficient, which is the sum of the real coefficient and the hysteresis effect:

$$(C_{m_q} + C_{m\dot{\alpha}})_{\text{apparent}} = (C_{m_q} + C_{m\dot{\alpha}})_{\text{real}} + \xi$$

where

$$\xi \equiv \left[\frac{\Omega d}{\pi}\frac{4m}{\rho Ad}\frac{I}{md^2}\frac{1}{C_{m_\alpha}{\alpha_0}^2}\int_{-\alpha_0}^{+\alpha_0} C_m d\alpha\right]_{\text{test hysteresis}} \qquad (3)$$

If this apparent coefficient is taken for the real coefficient and used to calculate the decay motion of a different size vehicle or one with a different mass distribution, the following results are obtained from Eq. (1):

$$\left(\frac{\delta\alpha}{\alpha_0}\right) = \frac{\pi}{\Omega d}\frac{\rho Ad}{4m}\left[C_{L_\alpha} - C_D - \frac{md^2}{I}\,(C_{m_q} + C_{m\dot{\alpha}})_{\text{real}}\right] - \xi\left(\frac{\pi}{\Omega d}\frac{\rho Ad}{4m}\frac{md^2}{I}\right)_{\text{scaled vehicle}} \qquad (4)$$

It is obvious that the predicted decay from this equation is not the same as would be predicted by using the real dynamic stability coefficient in Eq. (1). The difference lies in the quantity ξ, which is not necessarily the same for the two cases. However, if the dimensionless frequency Ωd is the same, then $(\rho Ad/4m)(md^2/I)$, which equals $-[(\Omega d)^2/2C_{m_\alpha}]$, is the same. If, in addition, the oscillatory amplitudes α_0 are the same for both cases, then, the scaled and test values of this term will be the same and the decay prediction of the scaled vehicle will be correct.

Although the case considered is simple, it nevertheless points out the fact that, as a minimum requirement for scaling, both the dimensionless frequency and the amplitude must be matched. If the motion is nonplanar, the problem of scaling becomes more complex. For motion that is completely circular, no flow hysteresis will occur, whereas planar motion creates a situation most susceptible to flow hysteresis. It seems obvious that, to match the hysteresis history, the scaled and test bodies must exhibit the same degree of nonplanar motion. For nonplanar motion, therefore, an additional parameter (or parameters) that describe the motion must also be matched for proper scaling. As a final note, one should not forget that, in addition to the motion parameters, the flow parameters, Mach number, Reynolds number, and enthalpy level must also be matched.

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Effects of Simulated Degrading Environment on Temperature Control Surfaces

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DATA exist concerning most of the recognized effects of 1) prelaunch atmosphere, 2) launch and staging, 3) charged particles in vacuum, and 4) ultraviolet irradiation in vacuum, upon the thermal characteristics of many commonly used materials. The particular design of the INTELSAT III satellite imposed an additional requirement upon the stability of one of its thermal control surfaces. Specifically, the plume radiance from an internally mounted rocket engine, used for synchronous orbit insertion, is such during firing that the adjacent surfaces are raised to a very high temperature for a short period of time. One of these surfaces (the spacecraft end cover) is then exposed to the ultraviolet and charged particle irradiation during the remainder of the spacecraft life. The temperature of this end cover is a function of its solar absorptance. In order to complete the thermal analysis of the spacecraft, it was necessary to determine the solar absorptance and emittance of this surface as a function of this sequence of events. It was also necessary to determine the plume absorptance of the coating during the motor firing in order to compute the end cover temperature. The heat input, E, to a surface by radiation is $E = \alpha G$, where G is the irradiation and α is the fraction absorbed.

The absorptance α may be determined by weighting the spectral absorptance over the spectral irradiance of the source. The power emitted by a surface equals $\epsilon\sigma T^4$. Monochromatically, the emittance of an opaque surface equals the absorptance. Hence, the total emittance can be obtained by weighting the spectral absorptance with the Planckian radiator function for the temperature of the surface. It is theoretically possible to have a surface with very low *solar* absorptance and very high 300°K emittance, as there is practically no spectral overlap of the solar irradiance and the 300°K Planckian radiator function. The same situation does not hold for a low plume absorptance and high emittance, since there is considerable overlap spectrally between the 1820°K plume temperature and 300°K and higher Planckian radiator functions.

There are two types of thermal control surfaces which are exposed to high temperatures during the spacecraft life. The first type, illustrated by the INTELSAT III end cover, is exposed to a high temperature and then must function as a low solar absorptance, high 300°K hemispherical emittance surface in sunlight. The second type operates at high temperature, and only the stability of its emittance at temperature (high or low depending upon the application) is important.

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Table 1 Preliminary list of materials with relative weighting factors

No.		Fabricability[a]	Initial Properties[b]	Resistance to Vac/Temp[b]	Resistance to uv[b]		α Solar	α Plume	ε, 300°K	ε, 750°K	Temperature and uv
1	Polyimide polymerized[c] directly on aluminized stainless steel foil	5	5	5	1	Initial	0.42	0.23	0.66	0.56	
						Heat only	0.49				750°K
						Heat only	0.58				800°K
2	Metallized polyimide[c] film bonded on stainless steel foil with adhesive	1	5	5	1	Initial	0.47	0.34	0.77	0.66	
						Heat only	0.65				800°K
3	Inorganic paint TRW MT6-2: potassium zirconium silicate	3	1	1	5	Initial	0.15	0.45	0.85	0.83	
						Heat only	0.20				800°K
						Final	0.33				+384 hr uv
						uv only	0.30				384 hr uv
4	Inorganic paint IITRI Z-93: zinc oxide pigmented potassium silicate	4	2	1	3	Initial	0.20	0.47	0.88	0.83	
						Heat only	0.30				800°K
						Final	0.33				+100 hr uv
						uv only					100 hr uv

[a] Higher number indicates more difficulty in fabrication.
[b] Higher number indicates less desirable properties.
[c] These samples were not exposed to uv irradiation since the increase in solar absorptance resulting from heat alone was so high.

In the study of the second group of surfaces (those for which solar absorptance values are not of interest), the subsequent ultraviolet irradiation is omitted because tests have indicated that emittance is usually unaffected by uv. The initial and final property measurements are usually done calorimetrically as part of the elevated temperature exposure.

Test Procedure and Materials

The following measurement sequence was used: initial spectral reflectance from 0.28 to 26 μ, solar absorptance and near-normal emittance using inspection devices, calorimetric hemispherical emittance. The environmental tests consisted of plume heating and subsequent ultraviolet irradiation. The property measurement sequence was repeated after the plume heating and again after the ultraviolet irradiation.

From the initial and final measurements, the desired properties were calculated. The solar absorptance was calculated by integrating the spectral absorptance over the solar irradiance. The plume absorptance was calculated by integrating the spectral absorptance over the plume irradiance. The hemispherical emittance was determined either directly from the calorimetric data or from the near-normal emittance which was obtained by integration of the spectral absorptance over the Planckian Radiator function. In many cases, the near-normal emittance was also determined on a model DB 100 infrared reflectometer (manufactured by Gier Dunkle Instrument Company).[1]

Several coatings had been selected initially as candidates for the low solar absorptance and high-emittance application. It was hoped that some compromise could be achieved amongst ease of manufacture, good thermal properties, and stability. Based on known data and our early tests, a preliminary list of materials was selected with relative weighting factors (see Table 1).

Details of Test

The initial solar absorptance was determined by measuring the spectral directional reflectance from 0.28 to 2.5 μ in a Beckman DK2A Spectrophotometer with integrating sphere of the Edwards et al. design.[2] This reflectance was integrated over the solar irradiance[3] to obtain solar absorptance.

The initial near-normal emittance was determined by measuring the spectral directional reflectance in a Gier Dunkle Heated Cavity Absolute Reflectometer.[4] This reflectance was integrated over the 300°K Planckian radiator function.

Some of the low α/ϵ coatings and all of the materials for which the α was not significant were tested calorimetrically. The hemispherical emittance was measured in a calorimetric hemispherical emittance device. This measurement is performed by determining the power density required to maintain the sample at equilibrium temperature in a water-cooled vacuum chamber. The emittance was measured at the temperature given. Four thermocouples were welded to the test sample. These thermocouples provide voltage and temperature data and thereby define the test area. Samples were heated electrically with alternating current. The current through the sample and the voltage across the test area were measured with a true RMS meter. The entire test assembly was within a water-cooled copper container with a high absorptance paint on the inside within an evacuated Pyrex bell jar. A system quite similar to the TRW Systems calorimetric hemispherical emittance device has been fully described by Abbott et al.[5]

The plume heating test was accomplished by two methods. Initially, small samples were heated by irradiation from a high

Table 2 Results for several high-temperature surfaces

Sample	Initial emittance at 1090°K	Time at elevated temperature		Final emittance at 1090°K
1) Haynes 25 alloy				
as received	0.43	60 min	1365°K	0.27
after heating	0.27	60 min	1475°K	0.42
2) Haynes 25 alloy				
sandblasted	0.51	60 min	1365°K	0.32
Columbium C 103 alloy				
uncoated	0.28	45 min	1700°K	0.34
uncoated	0.30	30 min	1920°K	0.14
Sample	**ϵ_i at 1090°K**	**Time at 1090°K**		**ϵ_f at 1090°K**
Bare nickel foil	0.16	60 min		0.16
Rhodium plated Nickel foil	0.15	60 min		0.15
Sample	**ϵ_i at 1400°K**	**Time at or above 1400°K**		**ϵ_f at 1400°K**
M-10[a,b]	0.79	120 min		0.77
L-20[a,b]	0.78	90 min		0.78
L-70[a,c]	0.89	60 min		0.85

[a] TRW proprietary process number.
[b] Aluminide coating on Columbium C103.
[c] Inorganic bonded sintered oxides on Columbium C103.

temperature source. Temperature control proved to be difficult, and direct electrical heating was used for later tests. The stainless steel substrate was heated by passing current through it.

The sample preparation and apparatus is the same as that used for the high-temperature calorimetric hemispherical emittance test. Thermocouples are placed along the sample, and a voltage is applied to the ends of the sample. The current is adjusted to give the desired temperature profile. All of these simulated plume heating tests were conducted within a vacuum of 10^{-6} torr or better.

Property measurements were made ex situ following the simulated plume heating tests and prior to subsequent tests.

Following the plume heating test, the samples were subjected to ultraviolet irradiation in vacuum. The measurement of spectral reflectance in the solar region was made in situ. This in situ apparatus is described elsewhere.[6] The design is such that any ultraviolet-induced reflectance changes can be measured without returning the sample to air. After the environmental tests had been conducted, the final property measurements were made.

Results

The results of the tests indicated that, for the low α/ϵ materials studied, there appeared to be no significant synergism between the plume heating and the ultraviolet irradiation. These results are shown in Table 1. Although not shown in the table, the polyimide film exhibited stable properties after exposure to 650°K in vacuo. Although not useable for this specific application, the material may be suitable under less severe conditions.

Table 2 shows the results for several high-temperature surfaces. For these materials, the solar absorptance is not important, and only the stability to the high-temperature exposure is significant. It is a characteristic of some alloys that a permanent change in emittance results from particular temperature exposure. This is shown in the first three samples in Table 2. The emittance of Haynes 25 decreases after heating to 1365°K where adsorbed surface gases are driven off. Above 1475°K, grain growth and/or other physical changes result in an increased emittance. The emittance of Columbium C103 increases slightly after 1700°K exposure, but decreases significantly after 1920°K exposure. The data for several high emittance coatings developed at TRW Systems are also given. These have been used where stable high emittance at high temperature is required.

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A Comparison of Approximate Analytical Techniques in Heat Conduction

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Introduction

A NUMBER of approximate analytical methods have been used in heat flow analysis.[1–9] Among these, the method of collocation,[2] Galerkin's method,[3] and the method of moments[2,4] (which includes the so-called heat balance integral)[5] are all classified as weighted residual techniques. A somewhat different technique, employing variational principles, was formulated by Biot over a decade ago[6] and has since been applied to nonlinear heat-conduction problems by Lardner.[7,8] A variational treatment of the heat equation was also developed by Washizu.[9] None of these approximate methods is restricted to heat-conduction problems alone. This Note compares the results of using each of them to solve two simple one-dimensional heat-conduction problems, in which a semi-infinite solid, initially at a uniform temperature, is suddenly exposed to 1) a constant surface temperature or 2) a constant surface flux, as the boundary condition.

Weighted Residual Methods

The one-dimensional heat-conduction equation for the case of constant material properties is

$$L(\theta) = \alpha\partial^2\theta/\partial x^2 - \partial\theta/\partial t = 0 \qquad (1)$$

where α is the thermal diffusivity defined as the ratio of the thermal conductivity to the product of density and specific heat $(k/\rho c_p)$, θ is the temperature above some reference temperature and $L(\theta)$ represents the operator for the heat equation. An approximate solution, θ_η, when substituted into Eq. (1) results in a residual or error term ϵ_η,

$$L(\theta_\eta) = \epsilon_\eta \qquad (2)$$

Suppose the residual is multiplied by a weighting function W_j, and averaged over the space ordinate. If the average is set equal to zero, the following form is obtained:

$$\int_x W_j L(\theta_\eta)dx = 0,\ j = 1,2,\ldots,\eta \qquad (3)$$

Thus, the solution is forced to satisfy Eq. (3) such that the operator on the approximate solution is orthogonal to the weighting factor over all values of the space ordinate. Generally, the form of θ_η is selected so that the boundary conditions are automatically satisfied, although this is not absolutely necessary. The form of θ_η will contain η arbitrary parameters q_η. These are evaluated by choosing a weighting function and applying Eq. (3). The result will be η equa-

Table 1 Comparison of results: constant surface temperature; assumed profile, $\theta = \theta_s(1 - x/\delta)^2$; result, $\delta = C(\alpha t)^{1/2}$

Approximate analytical method	C	$x/(\alpha t)^{1/2}$, where $\theta/\theta_s = 0.67$	Difference, %
Collocation at $x = \delta/2$	2.83	0.515	−14.2
Collocation at centroid, $x = \delta/4$	3.265	0.594	−1.0
Heat balance integral	3.46	0.630	+5.0
Galerkin method	3.17	0.577	−3.8
Biot method	3.36	0.611	+1.8
Exact solution	...	0.600	...

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